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Power corrections from small distances. *

F.V. Gubarev, M.I. Polikarpov,[a]and V.I. Zakharov[b]

[a] Institute of Theoretical and Experimental Physics,
B. Cheremushkinskaya, 25, 117259 Moscow

[b]Max-Planck Institut für Physik,
Föhringer Ring 6, 80805 München, Germany.

We review recent speculations on power like corrections in QCD which go beyond the standard Operator Product Expansion. Both the theoretical picture underlying these corrections and phenomenological manifestations are discussed in some detail.

1. Introduction

Power corrections have been discussed intensely in recent years, in particular, in review talks at the conferences in this series [1]. Thus, we would skip the general motivation and background. Instead, we will concentrate on one particular issue, that is speculations on hypothetical power corrections associated with short distances [2,3]. For the talk to be self contained, however, we will include also a very brief review of the standard picture based on the operator product expansion and underlying the QCD sum rules [4]. Naturally enough, in the standard picture we emphasize only the points which would be modified if there exist novel power corrections.

Thus, the outline of the talk as follows:

1. Standard picture.
2. Standard picture vs experiment.
3. Beyond the OPE.
4. Predictions, tests.

2. Standard picture.

Many of the ideas belonging now to the standard picture could in fact be traced back to a paper which is 51 years old, that is the paper by Casimir and Polder [5]. It is a pleasure to quote this paper which in fact is much more elaborated than the quotations may indicate. The first example of what we would interpret nowadays as a power correction can be understood by everybody. Namely, consider an e^+e^- pair at distance r placed into a center of a conducting cage of size L. Moreover, assume that $L \gg r$. Then the potential energy of the pair can be approximated as

$$V_{e\bar{e}}(r) \approx -\frac{\alpha_e}{r} + (const)\frac{\alpha_e r^2}{L^3}, \quad L \gg r \tag{1}$$

and the second term is a power correction to the Coulomb interaction. The derivation of (1) is of course straightforward in terms of the classical electrodynamics, since the correction is nothing else but the interaction of the dipole with its images. On the other hand, it can be derived also in terms of one-photon exchange. Moreover, to find the photon propagator one should find now the modes in the cage which are different from those in the empty space. The difference is of order unity at frequencies of order $\omega \sim 1/L$.

Now, we jump to QCD and conclude by analogy that the heavy quark potential in QCD looks at short distances as:

$$\lim_{r\to 0} V(r) \;=\; -\frac{c_{-1}}{r} + const \cdot \Lambda^3_{QCD} r^2, \tag{2}$$

where c_{-1} is calculable perturbatively as a series in α_s. Eq. (2) was derived first in Ref. [6]. Note the absence of a linear correction to the potential at short distances. The logic behind (2) is that we simply replace $L \to \Lambda^{-1}_{QCD}$ since the gluon propagator is modified by the infrared effects at $\omega \sim \Lambda^{-1}_{QCD}$.

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If one turns to consideration of bound states, then the quadratic correction in the potential (1) is washed out by the retardation effects [5]. Indeed, if T is the period of rotation of the charges and c is the speed of light then there might be not time enough to learn about the existence of the cage. Thus, if

$$T \cdot c \ \ll \ L, \tag{3}$$

then the retardation effects are crucial and one cannot use the potential (1). Instead, the shifts of the atomic levels in the cage are sensitive to a local characteristic of the non-perturbative fields which can be nothing else but $< 0|\mathbf{E}^2|0>_{non-pert}$. On dimensional grounds:

$$(\delta E)_{non-pert} \ \sim \frac{< 0|\mathbf{E}^2|0 >_{non-pert}}{m_e^3}, \tag{4}$$

where by $< 0|\mathbf{E}^2|0 >_{non-pert}$ one understands the difference between the average valued of $\mathbf{E}^2$ in one-photon approximation evaluated without and with the cage. Note that all the ultraviolet divergences cancel in this difference.

In fact $< 0|\mathbf{E}^2|0 >_{non-pert}$ was not introduced in [5] and only the numerical result was given which is possible since $< 0|\mathbf{E}^2|0 >_{non-pert} \sim L^{-4}$, with all the coefficients calculable. Thus, we brought Eq. (4) to a the modern form which first appeared within the Voloshin-Leutwyler picture for $Q\bar{Q}$ bound states [7]. In that case the corresponding density of the color field strengths, or the gluonic condensate $< 0|\alpha_s(G^a_{\mu\nu})^2|0 >$, cannot be calculated directly but can be extracted from independent data [4]. Thus, an analog of (4) looks as

$$\delta E_{n,l} \ = \ f_{n,l}\frac{< 0|\alpha_s(G^a_{\mu\nu})^2|0 >}{m_Q^4} + (pert.\ th.) \tag{5}$$

where $f_{n,l}$ encodes the dependence on the quantum numbers of the $Q\bar{Q}$ bound state and we included all the constants, such as powers of α_s, into the $f_{n,l}$. Further details and references can be found, e.g., in the review [8].

Another important prediction of the standard model is the absence of $1/Q^2$ corrections to the current correlation functions $\Pi_j(Q^2)$ at large Q^2 [4]. The correlation functions are defined as

$$\Pi_j(Q^2) \ = \ i\int e^{iqx}d^4x\langle 0|T\{j(x),j(0)\}|0\rangle, \tag{6}$$

where $q^2 \equiv -Q^2$ and $j(x)$ are local currents constructed on the quark and gluon fields and we suppress possible Lorenz indices.

Then, basing on the OPE, one can write at large Q^2:

$$\Pi_j(Q^2) \approx \ \Pi_j(Q^2)_{parton\ model} \cdot$$

$$\left[1 + \frac{a_j}{\ln[Q^2/\Lambda^2_{QCD}]} + \frac{b_j}{Q^4} + O((lnQ^2)^{-2}, Q^{-6})\right] \tag{7}$$

where the constants a_j, b_j depend on the channel, i.e. on quantum numbers of the current j. Terms of order $1/lnQ^2$ and Q^{-4} are representing the first perturbative correction and the gluon condensate, respectively. The absence of the linear correction in $1/Q^2$ is an analog of the absence of the linear correction to the potential at short distances (2).

To summarize, the standard picture assumes that the only change brought by non-perturbative fluctuations is the change in the ordinary Feynman graphs at virtual momenta of order Λ^{-1}_{QCD}. The power corrections parameterize these changes. Among the predictions of the standard picture are the absence of linear corrections to the $Q\bar{Q}$ potential at short distances $V(r)$ and of $1/Q^2$ corrections to the current correlation functions $\Pi_j(Q^2)$.

3. Standard picture vs experiment.

We are going now to quote a few results which, taken at face value, would rule the standard picture out. From the beginning, however, we would like to avoid any dramatic statement. It is all the more so that the authors of the papers quoted do not draw such conclusions themselves. In each case there could be specific problems, like subtraction of the perturbative contributions and we are not in position at all to analyze the data thoroughly. We would simply like to attract attention to the kind of measurements which can be crucial to check the standard picture.

(1) *The heavy quark potential at small r.*

As is mentioned above the standard picture predicts the absence of the linear correction at

$r \to 0$, see (2). The existing data are rather fitted to the form:

$$\lim_{r\to 0} V_{Q\bar{Q}}(r) \;=\; -\frac{c_{-1}}{r} + \sigma_0 \cdot r. \quad (8)$$

As for the numerical value of σ_0 it is conveniently expressed in units of σ_∞ where σ_∞ which is the string tension at large distances. Then:

$$\sigma_0 \;=\; (1 \;\div\; 5)\sigma_\infty. \quad (9)$$

Here the value $\sigma_0 \approx \sigma_\infty$ can be extracted as an estimate from the existing data for all r [9] while $\sigma_- \approx 5\sigma_\infty$ is the result of a very recent dedicated study of small distances [10]. The number is sensitive to the subtraction of the perturbative terms, see [11] and the Talk at this conference by Y. Schroeder.

A new and exciting perspective to measure the non-perturbative correction to the potential is provided by the so called P-vortices (for latest results and further references see, e.g., [12]). The P-vortices are particular non-perturbative field configurations observed on the lattice. The property which is crucial for us at the moment is that they dominate non-perturbative contributions and, in particular, reproduce the string tension σ_∞ as defined in terms of the static potential $V(r)$ at $r \to \infty$. Moreover, if one leaves the contribution of the P-vortices alone, then the Coulombic part is removed from $V(r)$ altogether. The non-perturbative potential $V(r)_{non-pert}$ defined now as arising from the P-vortices alone shows no change in the linear behavior down to r of order of the lattice size a [13]:

$$\lim_{r\to 0} V(r)_{non-pert} \;\approx\; \sigma_\infty \cdot r. \quad (10)$$

Finally, analytical studies of the fine structure of the charmonium levels also prefer strongly $\sigma_0 \approx \sigma_\infty$ [14].

(2) *Shifts of the $Q\bar{Q}$ atomic levels.*

On the lattice, one can measure the energy levels of the heavy $Q\bar{Q}$ systems as a function of the quark mass [2]. The latest measurements of this kind can be found in Ref. [15]. Using Eq. (5) one can interpret the results of the measurements in terms of the gluon condensate:

$$< 0|\alpha_s(G^a_{\mu\nu})^2|0 >_{exp} \;=\; f(m_Q) \quad (11)$$

where the standard picture corresponds to the function $f(m_Q) = const$. In reality, as the quark mass m_Q varies between ~ 5 and $\sim$50 GeV the function $f(m_Q)$ varies by a factor $15 \div 20$ [15].

(3) *Correlation functions.*

It has been known since long [16] that at least in some, so to say exceptional channels the sum rules (7) fail. Namely, one characterizes different channels by M^2_{crit} which is defined as the Euclidean mass scale where the asymptotic freedom is violated by the power corrections by 10%. Then the values of M^2_{crit} vary considerably in some cases:

$$(M^2_{crit})_{\rho-meson} \;\approx\; 0.6\, GeV^2, \quad (12)$$

$$(M^2_{crit})_{\pi-meson} \;\approx\; 2\, GeV^2, \quad (13)$$

$$(M^2_{crit})_{0^+ glueball} \;\approx\; 20\, GeV^2. \quad (14)$$

The estimates were obtained first by indirect means using theoretical input specific for the particular channel [16]. Moreover, the difference between the $\rho-$ and $\pi-$ channels is fully confirmed by the lattice measurements, see Ref. [17]. The standard picture, see Eq. (7), fails completely to reproduce the hierarchy of the mass scales. There are fits to the data based on parameterization of instanton contributions [18,16]. The model works well, with a notable exception of the σ-meson channel, see [17].

To summarize, the standard picture seems to disagree with a number of measurements. The best known case is the correlation functions in some channels (see point (3) above). In this case there were attempts to amend the situation by inclusion of the direct instantons. Measurements of the potential at short distances and the $Q\bar{Q}$ energy levels (points (1) and (2) above) are quite recent and have not been interpreted much.

4. Beyond the OPE. Compact photodynamics.

Soft corners of the phase space caught by the OPE is certainly not the only source of power cor-

[2] We are thankful to A. Leonidov for bringing our attention to this kind of measurements.

rections. As we will emphasize in this and next sections topological excitations could be also hidden behind the power corrections [2]. Whether this mechanism applies to QCD is an open question however.

The first example of a theory where the OPE does not work is very old in fact and goes back to the paper in Ref. [19]. However, since it has not been discussed in connection with the OPE, we will explain this example in some detail.

The action of the theory we are going to consider is very simple:

$$S \; = \; \frac{1}{4e^2}\int d^4x F^2_{\mu\nu} \tag{15}$$

where $F_{\mu\nu}$ is the Abelian field strength tensor, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The action (15) is that of free photons and at first sight nothing interesting can come out from this theory. In particular, if we introduce external static electric charges as a probe, their potential energy would be given by one-photon exchange without any corrections.

However, we shall see in a moment that, in a particular formulation, the theory admits also magnetic monopoles. Hence, a few preliminary words on the monopoles. Monopoles have magnetic field similar to the electric field of a charge:

$$\mathbf{H} \; = \; q_M \frac{\mathbf{r}}{4\pi r^3}. \tag{16}$$

Then the flux of the magnetic field through a surface surrounding the monopole is:

$$\Phi \; = \; q_M. \tag{17}$$

On the other hand, because of the equation $div\ \mathbf{H} = 0$ the magnetic flux is conserved. Thus, the magnetic monopole cannot exist by itself and one assumes that there is a string which is connected to the monopole and which brings in the flux. Moreover, to make the string invisible one assumes that the string is infinitely thin. Finally, to avoid the Bohm-Aharonov effect one imposes the Dirac quantization condition,

$$e\oint A_\mu dx_\mu \; = \; e\int \mathbf{H}\cdot d\mathbf{s} \; = q_M \cdot e \; = \; 2\pi \cdot n \tag{18}$$

Also, we ask for the energy (or action) associated with the Dirac string to vanish. Only then energies of the electric and magnetic charges are similar. We shall return to discuss the issue of the energy of the Dirac string later.

Now, the Dirac strings may end up with monopoles. The action associated with the monopoles is not zero at al but rather diverges in ultraviolet, since

$$\int \frac{d^3r}{8\pi}\mathbf{H}^2 \; \sim \; \frac{1}{e^2 a} \tag{19}$$

where a is a (small) spatial cut off. If the length of a closed monopole trajectory is L, then the suppression of such a configuration due to a non-vanishing action is of order

$$e^{-S} \; \sim \; \exp\ (-const\ L/e^2). \tag{20}$$

On the other hand, there are different ways to organize a loop of length L. This is the entropy factor. It is known to grow exponentially with L as $\sim \exp(\ const' L\ )$.

Thus, one comes to the conclusion [19] that at some $e_{crit} \sim 1$ there occurs a phase transition corresponding to condensation of the monopole loops. As a result, if external electric charges are introduced as a probe, their potential energy grows with distance, $V(r) \sim r$ and they are confined.

To complete the presentation we should explain how one should understand the theory (15) that it would imply a vanishing action for the Dirac string.

The crucial point is to define the theory by means of a lattice regularization. Then the action can be understood as a sum over plaquette actions:

$$\begin{aligned} S \; &= \sum \frac{1}{2e^2}\left(1 - \ Re\ \exp(i\oint A_\mu dx^\mu\right) \; = \\ &= \; \sum \frac{1}{2e^2}\left(1 - Re\ \exp(iF_{\mu\nu}d\sigma_{\mu\nu})\right) \; = \\ &= \; \sum \frac{1}{2e^2}\left(1-\cos(F_{\mu\nu}d\sigma_{\mu\nu})\right) \end{aligned} \tag{21}$$

where the sum is taken over all plaquettes (and note that no summation over the repeated indices μ, ν is understood). In the continuum limit one reproduces of course the action (15). However, from the intermediate steps it is clear that the action admits for a large jump in $F_{\mu\nu}$:

$$F_{\mu\nu} \; \rightarrow \; F_{\mu\nu} + 2\pi\delta(\sigma_{\mu\nu}) \tag{22}$$

where the δ-function on the surface is defined as $\delta(\sigma_{\mu\nu})d\sigma^{\mu\nu} = 1$ (no summation over μ, ν). The second term in Eq. (22) exactly corresponds to the Dirac string. Thus, the Dirac strings have no action in the lattice, or compact version of the $U(1)$ gauge theory.

Note that the UV scale $1/a$ is the only scale in the model. Thus, it rather exists only as a lattice theory. The confining potential sets in for $e > e_{crit}$ at all the distances and in this sense the situation does not imitate QCD.

5. Beyond the OPE: short strings.

We can introduce another scale, apart from the UV cut off, by adding a scalar field, i.e. by considering the Abelian Higgs model (AHM):

$$S = \int d^4x \big[\frac{1}{4e^2} F^2_{\mu\nu} + \frac{1}{2} |(\partial - iA)\Phi|^2 + \frac{1}{4}\lambda(|\Phi|^2 - \eta^2)^2 \big] \tag{23}$$

where λ, η are constants. The scalar field condenses in the vacuum, $\langle 0|\Phi|0\rangle = \eta$ and this represents a new scale. In particular, the physical vector and scalar particles are massive, $m_V^2 = e^2\eta^2, m_H^2 = 2\lambda\eta^2$. Moreover, to avoid the monopole condensation discussed in the preceding section we consider e small enough to neglect the dynamical monopoles.

Consider now two external magnetic charges separated by a distance r which are brought into the vacuum of the Abelian Higgs model. The problem is to find the potential energy $V(r)$ at the distances r much smaller than m_H^{-1}, m_V^{-1} [2]. The energy is determined in the classical approximation and, at first sight, the problem is not much of a challenge. The crucial point, however, that the problem is not yet properly formulated. Namely, one has to impose an extra boundary condition, that is vanishing of the scalar field along a mathematically thin line connecting the magnetic charges. This condition, a kind of topological one was formulated in its generality in Ref. [20] and implied in numerical studies of the problem (see, e.g., [21]). Moreover, the AHM is quite a common tool in phenomenological studies of QCD at large distances (see, e.g., [22]). The *short-distance* aspects of the model have only recently been emphasized [2].

The boundary condition $\Phi = 0$ along a line connecting the monopoles plays a crucial role. It is worth emphasizing therefore that this infinitely thin line is nothing else but the Dirac string connecting the external monopoles. As we discussed in the preceding section, the use of the lattice regularization implies a vanishing energy of the Dirac string in the perturbative vacuum. This is, in a way, a definition, how we understand the theory. Moreover, it is easy to realize that the Dirac string cannot coexist with $\Phi \neq 0$. Indeed, if the Dirac string would be embedded into a vacuum with $\langle\Phi\rangle \neq 0$ then its energy would again jump to infinity since there is the term $1/2|\Phi|^2 A_\mu^2$ in the action and $A_\mu^2 \to \infty$ for a Dirac string. Hence, $\Phi = 0$ along the string and it is our boundary condition. In other words, Dirac strings always rest on the perturbative vacuum which is defined as the vacuum state obeying the duality principle. Therefore, even in the limit $r \to 0$ there is a deep well in the profile of the Higgs field Φ. This might cost energy which is linear with r even at small r.

Thus, we are coming to the next question, whether this mathematically thin line realizes as a short physical string. Where by the physical string we understand a stringy piece in the potential, $\sigma_0 \cdot r$ at small r. In other words, we are going to see whether the stringy boundary condition implies a stringy potential. To get the answer one solves the classical equations of motion. The answer is, indeed,

$$\lim_{r\to 0} V(r) \;=\; -\frac{\alpha_M}{r} + \sigma_0 \cdot r \tag{24}$$

with a non-vanishing slope σ_0. It might worth emphasizing that this result is actually non-trivial. Indeed, naively, one could argue that the strong field near the monopoles “burns the Higgs field out” anyhow and the condition $\Phi = 0$ at small r is not important for this reason. In reality, however, this condition is manifested in $\sigma_0 \neq 0$. The form of the boundary condition finds its way to the final result for the energy and this can be viewed as a kind of analyticity.

The slope σ_0 depends smoothly on the value of m_V/m_H. For the purpose of orientation let us note that for $m_V = m_H$ the slope of the potential at $r \to 0$ is the same as at $r \to \infty$ That is, within error bars:

$$\sigma_0 \ \approx \ \sigma_\infty \tag{25}$$

where σ_∞ determines the value of the potential at large r.

The linear correction (25) to the potential violates the OPE. It is not the same obvious as in case of QCD, though, since now we do have an operator of $d = 2$, that is $|\Phi|^2$. However, a little more elaborated analysis than just counting the dimensions still demonstrates the violation of the OPE [2]. Qualitatively, it is indeed quite obvious that the topological boundary condition cannot be reproduced via the OPE but is imposed extra.

To summarize, existence of short strings has been proven in the classical approximation to the Abelian Higgs model. The linear piece in the potential at small distances reflects the boundary condition that $\Phi = 0$ along the straight line connecting the monopoles and violates the OPE. This is a manifestation of UV non-perturbative divergences in the energy of the Dirac string (see above).

6. Monopoles in QCD.

While monopoles are well known as classical solutions in theories with Higgs fields, they can also be defined in a pure topological way [20]. There is convincing evidence from the lattice simulations that monopoles as defined in the maximal Abelian projection condense in the QCD vacuum and dominate non-perturbative degrees of freedom, see, e.g., [23,24].

We refer the reader for any detail on the QCD monopoles to [23,24] and references therein and address here only one particular issue. Namely, from the experience with the Dirac strings we learn that pure topological defects in gauge theories are usually related to some singular field configurations and, generally speaking, infinite action. Only upon regularizing these UV non-perturbative divergences to zero one allows these fluctuations into theory and they may change the content of the theory completely. What are singular fields associated with the monopoles, if any? We would not be able to answer this question in full because of the lack of understanding the large-distance behavior, or infrared physics which is to provide a kind of mass for the charged gluons (as defined within the context of the Abelian projection [20]). However, an educated guess can be made about the small-distance singularities.

The idea is to exploit the classical monopole solution. As is well known the gauge field associated with the monopole solution can be represented as

$$A^a_\mu \ = \ \frac{f(r)}{gr^2}\epsilon^{a\mu b}r^b,\ \mu = 1,2,3;\ A^a_4 = 0. \tag{26}$$

The asymptotic behavior of the function $f(r)$ at small and large distances is crucial:

$$\lim_{r\to\infty} f(r) \ = \ 1, \quad \lim_{r\to 0} f(r) \ = \ 2 \ or \ 0 \tag{27}$$

(see, e.g., [25]) where by $r \to 0$ we still understand $r \gg m_H^{-1}$ since we are removing now the Higgs field altogether from the theory, to imitate the QCD. For the sake of estimates, the $r \to \infty$ asymptotic (27) can be used down to distances of order m_V^{-1} which in the QCD set up is to be replaced by Λ^{-1}_{QCD}. On the other hand, the small r asymptotic can also be stretched to $r \sim m_V^{-1}$ from below. It is worth emphasizing that the non-Abelian field strength tensor $G^a_{\mu\nu} \approx 0$ at small r. As a result from the Eq. (19) we would obtain estimate the integral $\int d^3r (G^a_{\mu\nu})^2$ as m_V^{-1}/g^2 which is correct.

Now start first with large distances and perform the gauge transformation which brings (26) to the Abelian form (see, e g., [23]). Then at small distances we generate a Dirac string which is directed along the third axis in the color space and along the x_3-axis in the space:

$$\begin{aligned} G_{\mu\nu} = -\sigma^3\ (\delta_{\mu,1}\delta_{\nu,2} - \delta_{\mu,2}\delta_{\nu,1})\cdot \\ \cdot\ \frac{2\pi}{g}\ \delta(x_2)\delta(x_1)\Theta(-x_3) \end{aligned} \tag{28}$$

Moreover the corresponding potential is singular at small r. For example, in the spherical coordinates, the θ-component of the potential is now given by:

$$A^{1+i2}_\theta \ = \ \frac{e^{\imath\phi}}{r}. \tag{29}$$

Also, if we start directly with the small r asymptotic $\lim_{r\to 0} f(r) = 2$ we would arrive at a potential which is singular at $r = 0$:

$$A^a_\mu \;=\; -\,i\;\mathrm{Tr}\,[\sigma^a\;(\vec{\sigma}\cdot\vec{n})\;\partial_\mu\;(\vec{\sigma}\cdot\vec{n})] \tag{30}$$

where $\vec{n}$ is the unit vector directed from the center of the monopole to the observation point and $\vec{\sigma}$ are the Pauli matrices.

If we take the potential (30) at face value, then we would conclude that the associated action is divergent in ultraviolet and infinite. Indeed, the potential (30) has a $\delta(r)$ type of singularity at $r = 0$. Thus, naively, the action is violently divergent:

$$\int ((G^a_{\mu\nu})^2 d^3x \;\sim \delta(r)$$

If there is a Higgs field then all the fields are smoothened at $r = 0$. Moreover, the contribution of the region $r \sim m_H^{-1}$ to the total action of the monopole solution is negligible. In this sense, the only function of the Higgs mass is to provide an ultraviolet regularization which allows for singular potentials (29),(30).

Now we come to the central point of UV regularization of the singular potentials in QCD. Since there is no Higgs field at all we should apply another regularization. The natural choice is of course the lattice regularization. Then we know already from the discussion of the compact $U(1)$ above that the Dirac string (28) is associated with no action. A new point which is specific for the monopoles is that the monopoles are placed into centers of the lattice cubes and, therefore, the lattice regularizes the $\delta(r)$ type of singularities (see above) to zero as well[3]. Thus the singular potential (30) is allowed on the lattice and brings no action, the same as for the classical solutions.

To summarize, we expect that the QCD monopoles are associated with potentials which are singular at the origin. The lattice UV regularization implies that such potential are allowed, however. A qualitative picture for the QCD monopoles would be that they are associated with singular potentials which bring no action at small r. Overlapping potentials of this type, however, are no longer solutions to $G^a_{\mu\nu} = 0$ and there is suppression because of the action associated with such an overlap. However, there is a gain in the entropy as well and one can expect that the balance is reached at $g^2 \sim 1$ since the entropy factors have no small parameter built in (see the discussion of the compact $U(1)$ above). Of course, our consideration is absolutely incomplete as far as there is no understanding of the physics at $r \sim \Lambda^{-1}_{QCD}$. In particular, as far as the $r \sim 0$ region is considered in isolation, the potential can be gauge rotated to zero.

[3]The detailed consideration will appear soon elsewhere.

7. A remark on the P-vortices.

Amusingly enough, the qualitative picture outlined in the conclusions to the preceding subsection might be subject to an experimental check by means of the P-vortices. While referring the reader for any detail about the P-vortices to a few recent articles [12] and references therein, here we just mention a few basic facts about the P-vortices.

One usually uses a specific gauge, namely, the gauge maximizing the sum

$$\sum_l |Tr\;U_l|^2 \tag{31}$$

where l runs over all the links on the lattice. The center projection is obtained by replacing

$$U_l \;\rightarrow\; sign\;(Tr\;U_l). \tag{32}$$

Each plaquette is marked either as (+1) or (-1) depending on the product of the signs assigned to the corresponding links. P-vortex then pierces a plaquette with (-1). Moreover, the fraction p of the total number of plaquettes pierced by the P-vortices and of the total number of all the plaquettes N_T, obeys the scaling law

$$p \;=\; \frac{N_{vor}}{N_T} \;\sim\; f(\beta) \tag{33}$$

where the function $f(\beta)$ is such that p scales like the string tension. Assuming independence of the piercing for each plaquette one has then for the center-projected Wilson loop W_{cp}:

$$W_{cp} = [(1-p)(+1) + p(-1)]^A \;\approx\; e^{-2pA} \tag{34}$$

where A is the number of plaquettes in the area stretched on the Wilson loop. Numerically, Eq. (34) reproduces the full string tension.

Now we argue that P-vortices correspond to large gauge potentials A^a_μ. Really the statement "large (or small) gauge potential" is obviously gauge dependent, and below we will discuss the maximal central gauge in which P-vortices are usually defined. Suppose that the plaquette is pierced by P-vortex, that is one or three links forming this plaquette have negative trace ($Tr\, U_l < 0$). It is easy to show that the links with negative trace correspond to large (infinitely large in the continuum limit) gauge potential.

Consider the link matrix, which correspond to the gauge potential A^a_l:

$$U_l = \exp\{i\frac{\sigma^a}{2}A^a_l\} = \cos\frac{\theta}{2} + i\vec{\sigma}\vec{n}\, sin\frac{\theta}{2} \qquad (35)$$

where $\theta = \sqrt{\sum_a |A^a_l|^2}$, and $n^a = A^a_l/\theta$ is the unit vector. If $Tr\, U_l < 0$ then $\cos\frac{\theta}{2} < 0$ or $\theta > \pi$. Recovering dimensional quantities we have: $\sqrt{\sum_a |A^a_l|^2} > \frac{\pi}{a}$, where a is the lattice spacing. Hence if P-vortices survive in the continuum then it means that in the continuum limit we have gauge potentials A^a_l of the order of the cutoff $\frac{1}{a}$. Thus P-vortices correspond to large gauge potentials just as field configuration (30), discussed in the previous section. Remarkable fact is that there exist a strong correlation of P-vortices and abelian monopoles, although these objects are defined in different gauges [26].

It is important that the P-vortex is constructed in a gauge-dependent way which makes it sensitive to large (i.e., non-accessible perturbatively) fields, but not large field strengths. This statement follows simply from the fact that P-vortices survive in the continuum limit. Let us note that practically all the papers which discuss P-vortices in the continuum (see [30,31] and references therein) use the formalism which corresponds to the assumption that the P-vortices pierce in fact plaquettes characterized by a negative sign of the Wilson loop. The idea that the P-vortex is sensitive to strong potentials rather than field strengths promoted in this talk has not been tested, to the best of our knowledge.

In the particular gauge P-vortices appear to be infinitely thin. Hence they prove in a sence that UV non-perturbative physics can be related to the physics of confinement. Note that P-vortices are not truly local objects: there definition involves the nonlocal gauge fixing procedure, in any other gauge P-vortex will be clearly nonlocal field configuration (for analogous discussion of gauge dependent operators in case of abelian projection see [29]). However, this relevance of the UV non-perturbative fields could well be an artefact of the gauge used. Violation of the OPE, on the other hand, would be a gauge independent proof of the relevance of the strong fields, as is explained in length in preceding subsections. From this point of view, the linear correction to the heavy quark potential would be most important. And, so far, the linear non-perturbative potential due to the P-vortices is observed numerically down to distances equal to a single lattice spacing.

In other words, Eq. (34) appears to work for all the distances. The violation of the standard picture is then traced to the fact that the P-vortices are infinitely thin. Indeed, what is crucial for the standard picture is a finite size of the non-perturbative fluctuations [28]. Note that in Ref [30] an attempt was made to modify (34) by assuming that the piercing of the plaquettes by the P-vortex is not random at distances below the thickness of the center vortex associated with the P-vortex. The resulting heavy quark potential immediately becomes quadratic at short distances, in accordance with the general theory. Thus, measurements of the potential seem to be of crucial importance.

To summarize, we argued that the P-vortices can be interpreted as evidence for essential role of the potentials which are singular in the continuum limit and survive on the lattice because of the lattice regularization. Measurements which demonstrate a linear potential due to the P-vortices at short distances indicate then that this role is not a mere artifact of the gauge used. In this sense confirmation of the existence of the linear correction would be of great importance.

8. Revisiting phenomenology.

In this section we come back to discuss (preliminary) evidence contradicting the standard picture and try to understand whether such evidence could be accommodated into the picture "beyond the OPE". It is worth emphasizing from the beginning that the breaking of the OPE was demonstrated theoretically only within the AHM [2]. The consideration of the QCD case outlined above by itself, i.e. without input of the data, is inconclusive. Thus, it is clear that at this moment phenomenology of the novel type power corrections can be based only on extra assumptions. It is amusing, nevertheless how well some simple assumptions work and we review briefly these assumptions.

Heavy quark potential at short distances.

It is well known that the infrared behavior of QCD is well described by the Abelian Higgs model with $m_V \approx m_H$, see, e.g., [32]. By assuming the analogy to be valid for the vacuum state also at short distances, one would predict $\sigma_\infty \approx \sigma_0$ (see Eq. (25). This prediction fits surprising well the potential induced by the P-vortices (see above).

Shifts of the $Q\bar{Q}$ atomic levels.

If, indeed, $\sigma_0 \approx \sigma_\infty$ then the predictions for the energy levels would be close to those obtained within the phenomenological Buchmuller-Tye potential [33] which incorporates $\sigma_0 \approx \sigma_\infty$:

$$(\delta E)_{non-pert} \approx (\delta E)_{Buchmuller-Tye}. \tag{36}$$

These predictions seem to be in much better shape in view of the lattice data [15] than the standard picture.

Correlation functions.

If the OPE is violated, there could arise corrections of order $1/Q^2$ in the r.h.s. of the Eq. (7):

$$\Pi(Q^2) \approx \Pi_j(Q^2)_{parton\ model} \cdot \tag{37}$$

$$\left(1 + \frac{a_j}{\ln[Q^2/\Lambda^2_{QCD}]} + \frac{b_j}{Q^4} + \frac{c_j}{Q^2} + ...\right) \tag{38}$$

Even if one accepts this assumption, it is far from being trivial to relate the new constants c_j to, say, linear term in the potential.

Qualitatively, however, one may hope that introduction of a tachyonic gluon mass at short distances would imitate the effect of the Λ^2_{QCD}/Q^2 corrections. Indeed, the stringy term in $V(r)$ at short distances can be imitated [34] by the Yukawa potential with a gluon mass λ:

$$\frac{4\alpha_s}{6}\lambda^2 \sim -\sigma_0. \tag{39}$$

While Eq (39) by itself is nothing else but a way to memorize the result for the potential at short distances, it is acquiring predictive power once one introduces the short distance tachyonic mass into one-loop graphs as well [3]. Amusingly enough, this bald assumption allows to explain paradoxes of the standard picture (see above) in a simple and unified way.

To begin with, phenomenologically, in the ρ-channel there are severe restrictions [35] on the new term c_j/Q^2:

$$c_\rho \approx -(0.03 - .07)\ GeV^2. \tag{40}$$

Remarkably enough, the sign of c_ρ does correspond to a tachyonic gluon mass (if we interpret c_ρ this way). Moreover, when interpreted in terms of λ^2 the constraint (40) does allow for a large λ^2, say, $\lambda^2 = -0.5 GeV^2$.

As for for the π-channel one finds now a new value of M^2_{crit} associated with $\lambda^2 \neq 0$:

$$M^2_{crit}(\pi - channel) \approx 4 \cdot M^2_{crit}(\rho - channel) \tag{41}$$

which fits nicely the Eqs. (13) and (14) above. Moreover, the sign of the correction in the π-channel is what is needed for phenomenology [16]. Fixing the value of c_π to bring the theoretical $\Pi_\pi(Q^2)$ into agreement with the phenomenological input one gets

$$\lambda^2 \approx -0.5\ GeV^2. \tag{42}$$

Finally, we can determine the new value of M^2_{crit} in the scalar-gluonium channel and it turns to be what is needed for the phenomenology, see Eq (15).

Further crucial tests of the model with the tachyonic gluon mass could be furnished with measurements of various correlators $\Pi_j(Q^2)$ on the lattice [3].

*To summarize,*in spite of the openly heuristic nature, the model with a short-distance tachyonic gluon mass works surprisingly well. Indeed, it resolves the long-standing paradoxes of the QCD sum rules without spoiling the successful predictions. At this moment, this model seems to be the best candidate for a "dogma", although this may change any time with arrival of new data. Let us also mention some particular dynamical schemes [36] which come close to imitation of the tachyonic gluon mass.

9. Conclusions

We have argued that a healthy phenomenology of the power corrections going beyond the OPE can be developed at this moment [3,2]. This observation by itself comes as a kind of surprise since the power corrections have been studied for more than 20 years. On the theoretical side, the violation of the OPE has been proven in case of the Abelian Higgs model [2]. In case of QCD, the results of the analysis outlined above are inconclusive.

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